Optimizing Missile Loads For U.S. Navy Combatants

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ABSTRACT

Although U.S. naval combatants carry a range of weapons, defensive and offensive missiles are their most formidable ones. U.S. Navy destroyers and cruisers carry their missiles in a vertical launch system embedded into their decks. Each ship is currently fitted with between 80 and 122 VLS cells and each cell holds one or more missiles. There are essentially nine major types of missile, some intended for defense of the ship and some for attacking other ships, submarines, aircraft, missiles, or land targets. A combatant ship deploys to sea with a fixed load of missiles and the mix of missile types influences how well she is prepared to carry out any of several anticipated missions. Any group of ships must be prepared for a variety of hostilities and does not get to choose which one it will actually encounter. We show how an available pool of missiles can be allocated among Navy combatants using an integer linear program to maximize their anticipated military effectiveness across many possible mission assignments in various operational plans.

"'Tis your noblest course.
Wisdom and fortune combatting together."

—Shakespeare, *Antony and Cleopatra*, III(13)

INTRODUCTION

U.S. Navy destroyers and cruisers carry a variety of weapons, but offensive and defensive missiles are their most formidable ones. While a combatant ship (or, simply, *combatant*) is pierside preparing to deploy, operational planners must choose the right missile load from those missiles available onshore in ready inventory in anticipation of encountering one or more missions at sea, drawn from a predefined set of potential missions. The combatant's deployment plan may suggest particular missions to anticipate, but overall we do not get to choose what an adversary decides to do, or even which adversary we will face. Accordingly, we allow for multiple warplans containing multiple mission requirements and load our ships as best able to deal with any one of them.

Most missiles on these ships are loaded in a vertical launch system (VLS), for example, the MK 41, as shown in Figure 1. Currently, each of these cruiser and destroyer combatants have VLS systems accommodating between 80 and 122 cells,

https://doi.org/1082598326225

APPLICATION AREAS:
Air and Missile Defense; Littoral Warfare and Regional Sea Control; Strike Warfare; Computing Advances in Military OR

OR METHODS:
Linear Integer Programming

Figure 1. Missile canister being loaded into a vertical launch system (VLS), and a 64-cell MK41 VLS configuration aboard the guided missile cruiser USS San Jacinto (CG56), which has a total of 122 missile cells aboard. This canister weighs about three metric tons. The missile it contains is mechanically fragile and contains fuel and high explosives, so it is not loaded or unloaded at sea and not without good cause. VLS exists in a number of variations, ranging from just four cells to as many as the 64 shown here. San Jacinto has 128 missile cells, 122 of which can contain missiles and six that have strikedown cranes used for missile inspection and maintenance. (U.S. Navy images and Jane's (2020), Naval Postgraduate School (2020)).

and each cell can hold one or more missiles depending on the missile type. Other Navy ships can also carry smaller numbers of missiles, principally for self-defense.

Presently there are nine major varieties of missile, some defensive and some offensive (see Table 1).

RGM-109 Tomahawk land attack missile (TLAM) is a long-range, subsonic cruise missile used for strike warfare missions. RGM-158C long-range antiship missile (LRASM) is a stealthy subsonic missile with autonomous ability to identify and attack a target. It is widely referred to as the AGM-158 because it can also be launched from U.S. aircraft. RIM-66 standard missile 2 (SM2) medium range (MR), including Blocks III, IIIA, and IIIB variants, is the primary ship-launched air defense and ship self-defense missile guided by semi-active radar or infrared sensor for terminal guidance. RIM-67 SM2 extended range (ER), now known as the RIM-156 SM2 Block IV variant, is a ship-launched defensive interceptor to provide extended range and improved high-altitude air defense capability. RIM-161 SM3 is a ship-launched defensive interceptor to counter short- to medium-range ballistic missile threats. It has been used to intercept a low-Earth orbit satellite. RIM-162 evolved sea sparrow missile (ESSM) is a ship-launched medium-range, semi-active homing missile for ship self-defense against enemy air and surface threats. It comes in four-packs, each of which fits into a single VLS cell. RIM-174 SM6 ER is a ship-launched, over-the-horizon air defense and surface warfare missile with an active seeker. RUM-139 antisubmarine rocket (ASROC) is a ship-launched antisubmarine torpedo.

U.S. cruisers and destroyers employ the AEGIS Weapon System, a centralized, automated, command-and-control and weapons control system, from detection to kill. The heart of the system is the AN/SPY, an advanced, automatic detect and track, multifunction phased-array radar. This high-powered radar is able to perform search, track, and missile guidance functions simultaneously, with a track capacity of more than 100 targets (U.S. Navy, 2020).

We will use the following Department of Defense definitions of military missions (Department of Navy, 2014, and Department of Defense, 2016):

- Air defense (AD): Defensive measures designed to destroy attacking enemy aircraft or missiles in the atmosphere, or to nullify or reduce the effectiveness of such an attack.
- Antisubmarine warfare (ASW): Operations conducted with the intention of denying the enemy the effective use of submarines.
- Protective escort: Defensive posture to deter, detect, prevent, and defend against attacks to high-value units.
- Strike: An attack to damage or destroy an objective or a capability.

Table 1. Missile types competing for space in VLS cells.

Missile	Cost $M	Mach	Range nm
RGM-109 Tomahawk	2	0.7	1,000
RGM-158C LRASM	4	0.8	300
RIM-66 SM2	0.4	3.5	70
RIM-67 SM2 ER	0.4	3.5	90
RIM-161 SM3	18	13	1,500
RIM-162 Sea Sparrow	1	4	30
RIM-174 SM6	5	3.5	130
RUM-139 ASROC	0.8	0.5	12

Notes: The three-letter missile designation describes the launch environment, mission symbol, and type of projectile. The first character, R, designates a ship-launched missile; the second character, G, I, or U, indicates a surface attack missile, aerial intercept missile, or underwater attack missile, respectively; the third character, M, indicates a guided missile. There are many variations of each of these missile types and the statistics shown are merely representative. The RIM-62 Sea Sparrow comes in four-missile packs that each fit in a single VLS cell. Some missiles are guided by controllers and some are completely autonomous. The Tomahawk, perhaps the best known, costs about $2 million, flies at 0.7 Mach (470 nm/hour), and has a range of 1,000 nautical miles; it can use Global Positioning System, inertial, terrain following, and digital optical scene matching guidance systems to autonomously find its target (Center for Strategic & International Studies, 2020; Department of Defense, 2020; Jane's, 2020; Naval Postgraduate School, 2020; Royal Australian Navy, 2020; Raytheon, 2020a, 2020b; U.S. Navy, 2019, 2020).

- Surface warfare (SUW): That portion of maritime warfare in which operations are conducted to destroy or neutralize enemy naval surface forces and merchant vessels.
- Theater ballistic missile defense (TBMD): Primarily defensive deployment of AD missiles to protect allies in a specific region or theater.

The combatant missile load-planning problem is currently addressed manually by the ship's administrative commander and shore-based weapons station personnel using basic Microsoft Excel 2016 spreadsheets to illustrate loadout recommendations. This method is labor intensive and provides no guidance about how or whether the plans might improve the ship's ability to contribute to various warplans given a different missile load. Often, a ship's missile load is based solely on which missiles are coming out of periodic depot maintenance, leaving operational fleet commanders with little influence. We contend that the U.S. Navy needs a mission-robust missile load plan exploiting any advantage we can discover that will perform well across a broad range of anticipated defensive and offensive actions. The solution to this problem is important: it is vital to wartime campaign success.

This missile assignment problem is further complicated by other restrictions, such as the cost, time, and risk of damaging missiles while moving them from inventory to a ship or to switch missiles between ships. We want to minimize unnecessary movement of missiles.

Further, some VLS-equipped ships cannot carry some missile types. Whereas all VLS ships can carry the right load for an escort mission, not all are able to carry TBMD loads. In some cases, there may be a substitute missile available when the better one for a particular mission is not. These missile substitutions are continually increasing as newer generations of missiles are developed; however, the VLS itself can become a constraint. Next-generation missiles have the potential to outgrow the size limitations of the current MK 41 VLS.

At any given time, some ships are in port and some are deployed. Some deployed ships are forward-deployed from ports overseas from the United States, operating in conjunction with deploying ships from ports in the United States. While these ships are deployed, planners are already preparing the missile load for the next set of ships that will relieve those currently in an area of responsibility. Forward-based fleet staffs desire to align the ship's missile loadouts with potential wartime tasking designated inside operational plans. The type and number of missiles she carries enable the ability for a single deployed ship to cover as many potential missions across these various operational plans. How her loadout fits into the currently deployed fleet's loadout is also a consideration in terms of which ship she may be replacing and which ships will be in theater during her deployment.

PREVIOUS RESEARCH

Various aspects of naval employment of missiles have been studied for decades. These range from single-ship missile-target pairing to multiship cooperative tactics. Models of "kill chain" events characterize target detection, tracking, missile assignment, firing, damage assessment, possible secondary launch, and so on. A representative sample from this literature follows.

Jarek (1994) used simulation to fill VLS cells for AD missile requirements. Two cases were explored: one in which combat air patrol was able to assist against the attack and one without such help.

Hughes (1995) introduced salvo equations: difference equations akin to continuous Lanchester differential models. Perhaps the most influential insight gained by his work is that when near-peer opponents engage, she who shoots first wins. Many other generalizations have appeared introducing deception, decoys, stochastic elements, etc.

Kuykendall (1998) used an integer linear programming model to optimize assignment of TLAMs to strike missions. The model takes into consideration the TLAM missile load on each ship, specific tasking, and geographic location. This work was later generalized by Newman et al. (2011) and is currently in use by the U.S. Navy.

Karasakal et al. (2011) present a missile defense optimization model for a cooperating naval task group. This expresses the sorts of interdependence that missile loadouts convey to combatants.

Dugan (2007) develops the Navy Mission Planner, an integer linear program to advise decision makers in assigning ships to missions with deadlines and identifying dependencies such as assigning a shotgun ship for air defense along with a ship assigned to TBMD for a given set of mission priorities. This model is run on a fictional case set around the Korean Peninsula.

Deleon (2015) introduces an integer linear program, the Navy Operational Planner, to help decision makers with maritime operational planning. His work explores our Navy's capability to accomplish missions as quickly as possible, rather than by pre-set mission accomplishment deadlines.

Wiederholt (2015) presents an integer linear program that determines the best missile load that can be achieved. Although his research supported 7th Fleet operational planners in the Western Pacific to include forward-deployed naval forces (FDNF) and deploying forces to the area, the problem can be extended to all U.S. fleet areas of responsibility. His work demonstrates the need for a better tool than current methods by showing the number of additional missions we can potentially fulfill.

Newman (2017) extends Wiederholt by substituting an open source Visual Basic for Applications (VBA) (Microsoft, 2020) evolutionary heuristic algorithm for licensed commercial packages General Algebraic Modeling System (GAMS) (GAMS, 2020) and CPLEX (IBM, 2020).

Brown and Kline (2021) present a Navy Mission Planner that considers operational restrictions to defend unarmed ships and includes logistics. Their planner integrates the full range of missile-dependent missions and dependencies among these.

Considerable additional references come from conference proceedings, but we restrict ourselves to open literature. Well, with one exception, we call out the Naval Postgraduate School MS in operations research thesis of Mike Mullen (1985). Admiral Mullen became the 17th Chairman of the Joint Chiefs of Staff in 2007. We have asked the AEGIS program manager on two occasions to declassify this thesis advised by our colleague Wayne Hughes, without success.

A MODEL TO SOLVE THIS PROBLEM: MISSILE LOAD PLANNER

The following is a linear integer program to assign ships and their missile loadouts to cover multiple possible mission assignments in different warplans. It is designed to assist the forward fleet operational planner in requesting ships be loaded out during deployment preparations with quantities of certain missiles to cover as many potential wartime missions as possible, while taking into account other ships' loadouts and potential mission assignments. Essentially, it ensures that during each deployment cycle, as many potential missions across various warplans may be filled by ships deployed by designating individual ship loadouts prior to deployment. An additional outcome of solving the loadout plan is the initial assignment of warplan missions to deployed ships, which may be helpful if the warplan must be executed.

Index Use [$\sim$Cardinality]

- $w \in W$: warplan [$\sim$3]
- $m \in M$: missions (alias m') [$\sim$10] (e.g., TBMD station)
- $d \in D$: deployment cycles [$\sim$2]
- $c \in C$: required mission ship classes (includes class "any") [$\sim$6]
- $s \in S$: individual ships [$\sim$25]
- $h \in H$: home ports [$\sim$2]
- $y \in Y$: missile types (alias y') [$\sim$9]
- $t \in T$: type of mission [$\sim$3]
- $r \in R$: risk level (including "high") [$\sim$2]
- r_m: risk level of mission m

Useful Tuples

Those marked with an asterisk (*) are derived and filtered from the others defined by data.

- $\{w,m\} \in WM$*: missions of warplan w [$\sim$10]
- $\{w,d,m\} \in WDM$: warplan-cycle-mission triples [10 x 10 x 2]
- $\{w,d,m,s\} \in WDMS$*: warplan-cycle-mission-ship 4-tuples [10 x 10 x 2 x 25]
- $\{s,d\} \in SD$: ship s deployment cycles
- $\{w,d,m,y\} \in WDMY$*: warplan-mission-cycle-missile 4-tuples [10 x 10 x 2 x 10]
- $\{m,m'\} \in MM'$*: missions m and m' are mutually exclusive (e.g., $t_m \neq t_{m'}$)
- $\{m,y,y'\} \in MYY'$*: missile type y can be substituted for type y'
- $\{w,d,m,s,y\} \in WDMSY$*: 5-tuple for missile requirements, or loading
- $\{w,d,m,s,y,y'\} \in WDMSYY'$*: 6-tuple for missile loading with substitutions
- $\{m,c\} \in MC$: indicates mission m can be completed by ship class c
- $\{s,y\} \in SY$: indicates ship s can accommodate missile type y

Given Data [Units]

- $priority_m$: priority of mission m [penalty]
- $ships\_req_m$: ships required by mission m [ships]
- $ships\_short\_pen_m$: ship shortfall penalty for mission m [penalty/ship]
- $missiles\_desired_{m,y}$: desired type y missiles on each ship for mission m [missiles]
- $missiles\_minimum_{m,y}$: minimum missiles on each ship of type y for mission m [missiles]
- $missile\_short\_pen_{m,y}$: missile shortfall penalty for mission m, type y [penalty/missile]
- vls_cells_s: number of VLS cells on ship s [cells]
- $missile\_inventory_y$: number of type y missiles in inventory [missiles]
- $missiles\_per\_cell_y$: number of type y missiles in a VLS cell [missiles per cell]
- $under\_pen_y$,$over\_pen_y$: penalty for disproportionate spread of missile type y among ships carrying these for each mission [penalty]
- $min\_missile\_load_{s,y}$: minimum ship s type y missiles that must be carried for that ship to be assigned any mission [missiles]

- $risk\_missile\_load_{s,y}$: ship s, type y missiles carried in addition to minimum missile load if any mission assigned to this is high risk [missiles]
- $alt\_missile\_pen_{m,y,y'}$:penalty for substituting type y' for y in mission m [penalty/missile]
- $loadout_{s,y}$: ship s load of missile type y prior to optimization [missiles]
- $change\_pen$:penalty for adjusting prior loadout [penalty/missiles]

Decision Variables [Units]

- $ASSIGN_{w,d,m,s}$: assign ship s to warplan w deployment cycle d mission m [binary]
- $MISSION_{w,d,m}$: plan w, cycle d, mission m commitment [binary]
- $COMMIT_{w,d,m,s,y,y'}$: plan w, cycle d, mission m, ship s, want type y, commit type y' [missiles]
- $MISSILE\_SLACK_{w,d,m,y}$: plan w, cycle d, mission m, type y missiles short of desired number [missiles]
- $SHIPS\_SHORT_{w,d,m}$: plan w, cycle d, mission m, elastic variable (e.g., Brown and Rosenthal, 2008) for ship shortages on mission [ships]
- $MISSILES\_SHORT_{w,d,m,y}$: plan w, cycle d, mission m, elastic variable for type y missile shortages on missions [missiles]
- $LOAD_{s,y'}$: ship s load of missile type y' [missiles]
- $RISK\_MISSILES_{s,y'}$: carried by ship assigned high-risk mission(s) [missiles]
- $UNDER_{w,d,m,s,y}$,$OVER_{w,d,m,s,y}$: elastic variable for inequitable missile loads [fraction]
- $CHANGE_{s,y'}$: number of y' missiles changed in VLS cells of ship s [missiles]
- $DEPLOY_s$: indicator that ship s is deployed [intrinsically binary]
- $DEPLOY\_WAR_{w,s}$: indicator that ship s is deployed in warplan w [intrinsically binary]

Formulation

$$\begin{aligned}
\text{MIN} \; & - \sum_{\{w,d,m\}\in WDM} priority_m MISSION_{w,d,m} \\
& + \sum_{\{w,d,m,s,y,y'\}\in WDMSYY'} alt_missile_pen_{m,y,y'} COMMIT_{w,d,m,s,y,y'} \\
& + \sum_{\{w,d,m\}\in WDM} ships_short_pen_m SHIPS_SHORT_{w,d,m} \\
& + \sum_{\{w,d,m,y\}\in WDMY} missiles_short_pen_{m,y} MISSILES_SHORT_{w,d,m,y} \\
& + \sum_{\{w,d,m,s,y\}\in WDMSY} under_pen_y UNDER_{w,d,m,s,y} \\
& + \sum_{\{w,d,m,s,y\}\in WDMSY} over_pen_y OVER_{w,d,m,s,y} \\
& + \sum_{\{s,y\}\in SY} change_pen CHANGE_{s,y},
\end{aligned} \tag{1}$$

$$\text{s.t. } ASSIGN_{w,d,m,s} + ASSIGN_{w,d,m',s} \le 1 \quad \forall \{w,d,m,s\} \in WDMS, \; \{w,d,m',s\} \in WDMS, \; |\{m,m'\} \in MM', \tag{2}$$

$$MISSION_{w,d,m} \ge ASSIGN_{w,d,m,s} \quad \forall \{w,d,m,s\} \in WDMS, \tag{3}$$

$$\sum_{s|\{w,d,m,s\}\in WMDS} ASSIGN_{w,d,m,s} + SHIPS\_SHORT_{w,d,m} = ships\_req_m MISSION_{w,d,m} \quad \forall\{w,d,m\} \in WDM, \tag{4}$$

$$RISK\_MISSILES_{s,y} \geq \left(min\_missile\_load_{s,y} + risk\_missile\_load_{s,y}|_{r_m='high'}\right) ASSIGN_{w,d,m,s} \quad \forall\{w,d,m,s,y\} \in WDMSY, \tag{5}$$

$$\sum_{y'|\{w,d,m,s,y,y'\}\in WDMSYY'} COMMIT_{w,d,m,s,y,y'} \leq missiles\_desired_{m,y} ASSIGN_{w,d,m,s} \quad \forall\{w,d,m,s,y\} \in WDMSY, \tag{6}$$

$$LOAD_{s,y'} \geq \sum_{\{d,m,y\}|\{w,d,m,s,y,y'\}\in WDMSYY'} COMMIT_{w,d,m,s,y,y'} + RISK\_MISSILES_{s,y''} \quad \forall w \in W, \{s,y'\} \in SY, \tag{7}$$

$$\sum_{\{s,y'\}|\{w,d,m,s,y,y'\}\in WDMSYY'} COMMIT_{w,d,m,s,y,y'} + MISSILES\_SHORT_{w,d,m,y} + MISSILE\_SLACK_{w,d,m,y} \geq missiles\_desired_{m,y} MISSION_{w,d,m} \quad \forall\{w,d,m,y\} \in WDMY, \tag{8}$$

$$\sum_{\{s,y\}\in SY} LOAD_{s,y}/missiles\_per\_cell_y \leq vls\_cells_s \forall s \in S, \tag{9}$$

$$\sum_{\{s,y\}\in SY} LOAD_{s,y} \leq missile\_inventory_y \quad \forall y \in Y, \tag{10}$$

$$\sum_{y'|\{w,d,m,s,y,y'\}\in WDMSYY'} COMMIT_{w,d,m,s,y,y'} + UNDER_{w,d,m,s,y} - OVER_{w,d,m,s,y} = \left(missiles\_desired_{m,y}/ships\_req_m\right) ASSIGN_{w,d,m,s} \quad \forall\{w,d,m,s,y\} \in WDMSY, \tag{11}$$

$$CHANGE_{s,y} \geq +\left(LOAD_{s,y} - loadout_{s,y}\right) \quad \forall\{s,y\} \in SY \;|\; \sum_{y|\{s,y\}\in SY} loadout_{s,y} > 0, \tag{12}$$

$$CHANGE_{s,y} \geq -\left(LOAD_{s,y} - loadout_{s,y}\right) \quad \forall\{s,y\} \in SY \;|\; \sum_{y|\{s,y\}\in SY} loadout_{s,y} > 0, \tag{13}$$

$$DEPLOY_s \geq ASSIGN_{w,d,m,s} \quad \forall\{w,d,m,s\} \in WDMS, \tag{14}$$

$$DEPLOY_s \leq \sum_{m|\{w,d,m,s\}\in WDMS} ASSIGN_{w,d,m,s} \quad \forall\{w,d,s\} \in WDS, \tag{15}$$

$$DEPLOY\_WAR_{w,s} \geq ASSIGN_{w,d,m,s} \quad \forall\{w,d,m,s\} \in WDMS, \tag{16}$$

$$DEPLOY\_WAR_{w,s} \leq \sum_{m|\{w,d,m,s\} \in WDMS} ASSIGN_{w,d,m,s} \quad \forall \{w,d,s\} \in WDS, \tag{17}$$

$$\begin{array}{ll}
ASSIGN_{w,d,m,s} \in \{0,1\} & \forall \{w,d,m,s\} \in WDMS, \\
MISSION_{w,d,m} \in \{0,1\} & \forall \{w,d,m\} \in WDM, \\
COMMIT_{w,d,m,s,y,y'} \in Z^{+} & \forall \{w,d,m,s,y,y'\} \in WDMSYY', \\
MISSILE_SLACK_{w,d,m,y} \in [0, \quad missiles_desired_{m,y} - missiles_minimum_{m,y}] & \forall \{w,d,m,y\} \in WDMY, \\
SHIPS_SHORT_{w,d,m} \geq 0 & \forall \{w,d,m\} \in WDM, \\
MISSILES_SHORT_{w,d,m,y'} \geq 0 & \forall \{w,d,m,y'\} \in WDMY, \\
LOAD_{s,y'} \geq 0 & \forall \{s,y'\} \in SY, \\
RISK_MISSILES_{s,y'} \geq 0 & \forall \{s,y'\} \in SY, \\
UNDER_{w,d,m,s,y}, OVER_{w,d,m,s,y} \geq 0 & \forall \{m,d,m,s,y\} \in MDMSY, \\
CHANGE_{s,y'} \geq 0 & \forall \{s,y'\} \in SY, \\
DEPLOY_{s} \geq 0 & \forall s \in S, \\
DEPLOY_WAR_{w,s} \geq 0 & \forall w \in W, s \in S.
\end{array} \tag{18}$$

DISCUSSION

The objective (1) is minimized, so it accounts for rewards using a minus sign for prioritized mission accomplishment and deducts penalties using a plus sign for violating policies that cannot (or the planner decides should not) be satisfied. These include a deduction for substituting a less capable missile for a preferred one, a penalty for not assigning the desired number of ships to a mission, one for not assigning the desired number of missiles to a mission, and penalties for not balancing missile commitments among ships assigned to a mission. These penalties may result from violating optional model features introduced in the following constraints. Each constraint (2) restricts a ship from performing mutually exclusive missions. Each constraint (3) signals a mission accomplishment if any ship is assigned to this mission. Each constraint (4) provides the required number of ships for a mission, or accounts for any (penalized) shortfall. Constraint (4) has been modified from the original Wiederholt formulation to ensure that if a mission is not committed to, then there is no ship penalty associated with that shortage. However, the model informs the naval planner when a mission is not assigned to a ship. Each constraint (5) reckons whether a ship needs extra defensive missiles due to the risk level of missions assigned to it. Each constraint (6) commits a number of a required missile type, or an acceptable substitute type to fulfill an assigned mission. Each constraint (7) reckons the number of missiles of some type that are to be loaded on a ship. Each constraint (8) reckons whether the required number of missiles has been loaded, or accounts for a (penalized) shortfall. Each constraint (9) limits the number of missiles that can be loaded into the VLS of a ship. Each constraint (10) limits the number of a type of missile to the total in available inventory.

Each (optional) constraint (11) requires that a type of missile be loaded proportionately on each ship participating in a mission, or a (penalized) imbalance is reckoned. This is to preserve residual firepower in case a shooter is unable and avoid a single point of failure. Each constraint (12) and its twin (13) (optionally) reckon the persistent positive difference (Brown et al., 1997) between a pre-existing VLS loadout and the one being prescribed by the model. This positive difference is penalized in the objective function in order to reduce unnecessary "turbulence" between legacy loadouts and their optimal revisions, but could just as well be limited numerically by ship and by missile type if it is anticipated that there will be limited pierside time to make changes. Constraints (14) and (15) are, together, optional. Each constraint (14) sets an indicator that a ship has been assigned a mission in some deployment cycle of some warplan by the naval planner.

Each constraint (15) assures that a deployed ship is assigned at least one mission in each deployment cycle of each warplan.

Constraints (16) and (17) are, together, optional and are subsumed if constraints (14) and (15) are invoked. Each constraint (16) sets an indicator that a ship has been assigned a mission in some deployment cycle of some warplan. Each constraint (17) assures that a deployed ship is assigned at least one mission in each deployment cycle of each warplan to which it has been assigned a mission. Constraint (18) defines decision variable domains.

This optimization model discovers a best single VLS loadout for each ship, where "best" is in the opinion of the planner balancing various penalties as they arise. Each ship is given one loadout regardless of whether she is a deployer making one deployment cycle or a FDNF ship in theater for two deployment cycles or longer. The solution also advises the best ship-to-mission pairing for some number of warplans. The loadouts provide the best solution to be prepared regardless of which warplan is realized. The warplans are not weighed by any prior probability, though that would be easy to do. Planners retain control to set a ship's loadout manually. The optimization model will account for this fixed loadout and assign the remaining ship loadouts and mission assignments to best cover the required missions.

COMPUTATIONAL EXPERIMENTS

Wiederholt (2015) verified his VLS load planner with many experiments. Our team sought advice from fleet personnel with experience and current planners. We settled on a test model with 23 combatants (five cruisers [CGs], 17 destroyers [DDGs], and one DDG 1000), nine missile types, and two fictional warplan scenarios: one with 22 missions and the other with 30. Scenarios analyzed included assigning missions the best we can with fixed missile loads for all ships; then with fixed loads for all ships deployed in theater, but allowing deployers from the United States for "cycle 2" to re-optimize missile loads; then with all ships optimally loaded; then with a restricted missile inventory; and finally with some ships not yet equipped to accommodate the then relatively new SM6 (see Table 2).

Wiederholt reports his integer linear program to plan all of these VLS loadouts has 9,300 constraints and 19,000 variables, 9,000 of which are integers. On a Lenovo W530 portable workstation with 32 gigabytes of random access memory and eight processors, GAMS version 31.1 (GAMS, 2020) and CPLEX version 12.10 (IBM, 2020), it generates and solves this problem to an integer tolerance of 10% with CPLEX in 10 minutes. The GAMS interpreter and CPLEX solver require 75 megabytes of random access memory for this model. Model performance was reliable and solutions intuitive: the more degrees of freedom we are granted to assign missiles optimally, the more missions we can accomplish.

We have since been able to reduce optimization times to about 10 seconds, with an integer tolerance of 10%, but an achieved tolerance, case after case, of exact integer solutions.

The optimization model does not seem to be particularly sensitive to the number of alternate warplans, missile types, or restrictions on re-loading missiles. The number of ships is an exogenous constant we cannot control much without new shipbuilding appropriations. We are comfortable we can accommodate any reasonable policy restrictions of missile loading, including those

Table 2. Generic families of planning test problems (from Wiederholt, 2015).

Family
1. Fixed missile loadout
2. Fixed and flexible missile loadout
3. Flexible missile loadout for all ships in all cycles
4. Changing warplan preferences
5. Reduced missile inventory
6. Ships unable to accommodate certain missiles

Note: Each scenario presents a series of planner challenges requiring adjusting penalties to try to address flaws and may require scores of planning runs. These problems helped shape the design of the graphical user interface to make it more intuitive.

imposed by replacement schedules required for periodic depot maintenance, swaps between ships pierside at amenable times for such, etc.

Encouraged by these results, we built a planner interface. A Microsoft Excel (2020a) workbook with added VBA (Microsoft, 2020b) tools controls Missile Load Planner (MLP).

Operational data is classified and most Navy operational planners must use computers that lack GAMS or CPLEX. Fortunately, the volume of data required for missile load planning is modest, so transferring scenarios to electronically isolated analysis workstations is not too hard.

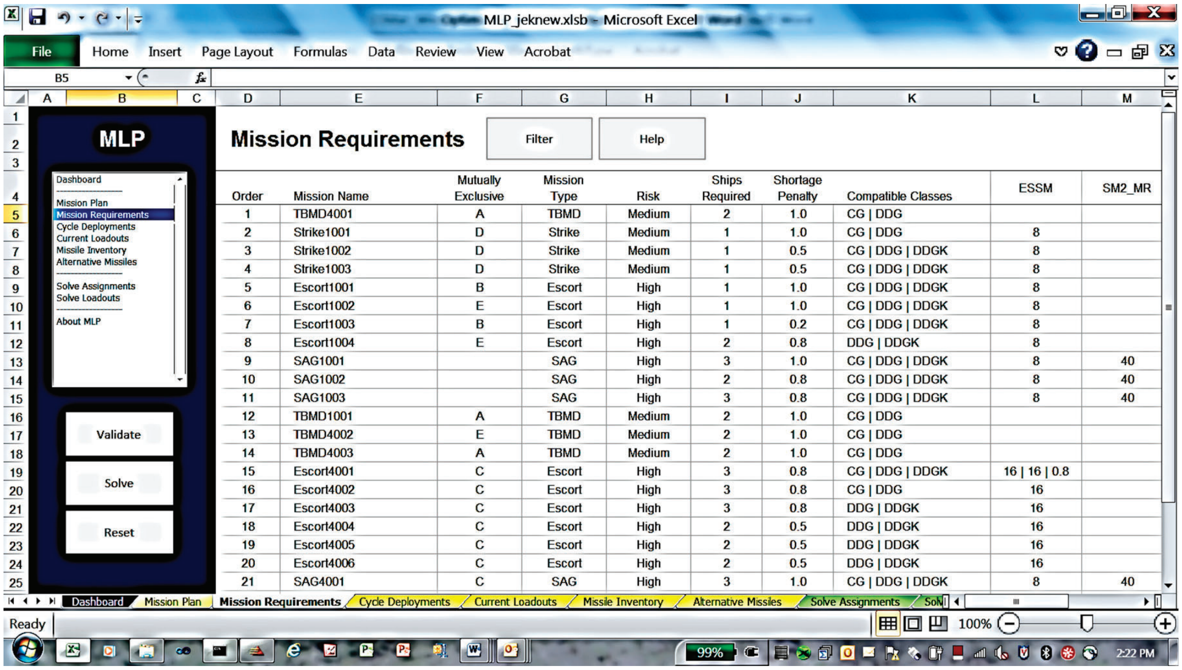
However, the standard Navy computers, even with their impoverished endowment of approved software, do have Microsoft Office, which we have used to create a user interface with its spreadsheet Excel and a heuristic solver with its VBA. A simple VBA procedure writes (comma-separated-value) csv files with set element names (synonym: identifiers) and associated data values.

We have two considerations when trying to replace a powerful modeling language such as GAMS with heuristic VBA code. One is replacing model set element names with numeric indexes (synonym subscripts) directly applicable by iterative control structures. Each set element name is a character string typically less than 64 characters (bytes) in length, with the first character an alphabetic one and following characters alphanumeric (perhaps with some additional special characters permitted). Each numeric index is an integer (=0, 1, 2, . . .) four bytes in length. We use a hash function (e.g., Bentley, 2000) to create a symbol table (e.g., Bentley, 2016) for each unary set, where the ordinal position of each element name in its symbol table is its index. While creating a symbol table, each element name is checked for syntactic correctness and uniqueness. When referring to the symbol table, we use an element name to discover its index.

For k-tuple sets (those with elements consisting of some number, or set dimension, $k > 1$ of names), the name of each k-tuple set element is converted to its index and a symbol table is initially built storing these tuples of indexes and subsequently accessed to retrieve for each tuple of names their respective indices. Not much code is required to convert character strings to hash addresses, check in a hash table and its symbol table for a vacancy, or a match, and either create a new entry or return the index(es) associated with the set element name(s).

The second consideration is the order in which one defines sparse k-tuple-sets (i.e., tuple sets with cardinality far fewer than the product of the cardinality of each of the k sets in the tuple). An example here is the 6-tuple $WDMSYY$. We use the term "derived" index tuple to stress the importance of starting with unary sets and building k-tuple sets increasing in k and with reference to any relationship known among the k elements. For example, suppose sets $\{a,b\} \in AB$ and $\{b,c\} \in BC$ are defined and that each is sparse (i.e., the cardinality of AB is far less than the product of the cardinalities of a and b). If we need to build set $\{a,b,c\} \in ABC$ we are well advised to do so only after establishing sets AB and BC and while filtering $\{a,b,c\}$ for simultaneous existence in sets AB and BC. When we started VBA work, just generating the sets and their tuples for a Wiederholt base case required 18 minutes on a Lenovo P50 workstation; a lot of polish introducing derived index sets, some only created for intermediate use to advise membership in other sets of higher dimension, reduced this to five seconds.

The Excel interface organizes scenario data in worksheets for mission plan, mission requirements (shown in Figure 2), cycle deployments, current loads, missile inventory, and alternate missiles. A validate button runs a macro that diagnoses any missing or inconsistent data. A solve button runs a macro that generates and exports csv format files (e.g., Microsoft 2020c), and either uses GAMS (2020) that calls CPLEX (IBM, 2020), or generates and heuristically solves the model in all-open-source VBA. From either path, MLP subsequently imports csv format solution files. After the optimization software returns the (near-) optimal solution, our dashboard displays loads and assignments in a "solve assignments" worksheet (Figure 3) with rows showing warplan, mission name, cycle1 and cycle2 ship assignments; and a "solve loadout" worksheet (Figure 4) with rows showing ship, VLS cells available, VLS cells loaded, the numbers of each missile type loaded, and how much of a change this is from the load prior to our optimization.



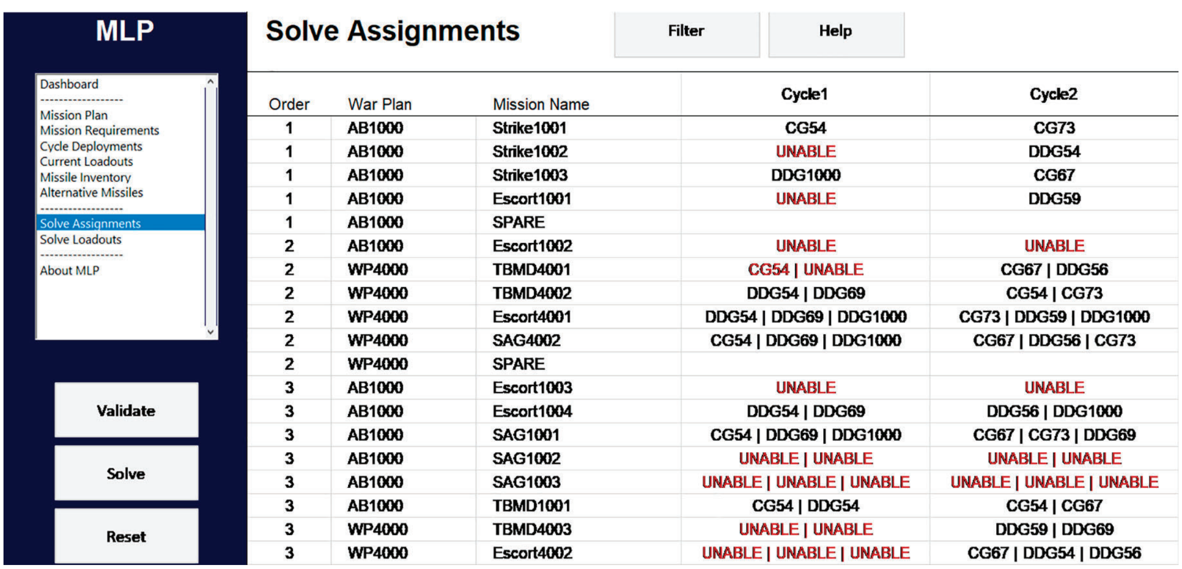
Mission Requirements

Order	Mission Name	Mutually Exclusive	Mission Type	Risk	Ships Required	Shortage Penalty	Compatible Classes	ESSM	SM2_MR
1	TBMD4001	A	TBMD	Medium	2	1.0	CG | DDG		
2	Strike1001	D	Strike	Medium	1	1.0	CG | DDG	8	
3	Strike1002	D	Strike	Medium	1	0.5	CG | DDG | DDGK	8	
4	Strike1003	D	Strike	Medium	1	0.5	CG | DDG | DDGK	8	
5	Escort1001	B	Escort	High	1	1.0	CG | DDG | DDGK	8	
6	Escort1002	E	Escort	High	1	1.0	CG | DDG | DDGK	8	
7	Escort1003	B	Escort	High	1	0.2	CG | DDG | DDGK	8	
8	Escort1004	E	Escort	High	2	0.8	DDG | DDGK	8	
9	SAG1001		SAG	High	3	1.0	CG | DDG | DDGK	8	40
10	SAG1002		SAG	High	2	0.8	CG | DDG | DDGK	8	40
11	SAG1003		SAG	High	3	0.8	CG | DDG | DDGK	8	40
12	TBMD1001	A	TBMD	Medium	2	1.0	CG | DDG		
13	TBMD4002	E	TBMD	Medium	2	1.0	CG | DDG		
14	TBMD4003	A	TBMD	Medium	2	1.0	CG | DDG		
15	Escort4001	C	Escort	High	3	0.8	CG | DDG | DDGK	16 | 16 | 0.8	
16	Escort4002	C	Escort	High	3	0.8	CG | DDG	16	
17	Escort4003	C	Escort	High	3	0.8	DDG | DDGK	16	
18	Escort4004	C	Escort	High	2	0.5	DDG | DDGK	16	
19	Escort4005	C	Escort	High	2	0.5	DDG | DDGK	16	
20	Escort4006	C	Escort	High	2	0.5	DDG | DDGK	16	
21	SAG4001	C	SAG	High	3	1.0	CG | DDG | DDGK	8	40

Figure 2. Mission Load Planner mission requirements page. Row 15 shows mission Escort4001, a mission that is mutually exclusive with other (escort) missions also labeled C. This is an escort mission that is high risk, requiring some combination of three ships of cruiser or destroyer classes, with participants loaded with 16 (four VLS cells) of extra self-defense ESSM Sparrow missiles each. The "Validate" button checks all data for consistency and the "Solve" button can invoke any of our solvers.

RESULTS

In practice, cost, risk of damage, and time limitations for missile removal and replacement pose the most difficult manual decisions; MLP takes guidance from the planner and quickly solves this puzzle. The MLP plan has visibility of all the candidate ship loadouts, and balances all of them at once against potential mission assignments. One can assess the tradeoff between potential mission



Solve Assignments

Order	War Plan	Mission Name	Cycle1	Cycle2
1	AB1000	Strike1001	CG54	CG73
1	AB1000	Strike1002	UNABLE	DDG54
1	AB1000	Strike1003	DDG1000	CG67
1	AB1000	Escort1001	UNABLE	DDG59
1	AB1000	SPARE		
2	AB1000	Escort1002	UNABLE	UNABLE
2	WP4000	TBMD4001	CG54 | UNABLE	CG67 | DDG56
2	WP4000	TBMD4002	DDG54 | DDG69	CG54 | CG73
2	WP4000	Escort4001	DDG54 | DDG69 | DDG1000	CG73 | DDG59 | DDG1000
2	WP4000	SAG4002	CG54 | DDG69 | DDG1000	CG67 | DDG56 | CG73
2	WP4000	SPARE		
3	AB1000	Escort1003	UNABLE	UNABLE
3	AB1000	Escort1004	DDG54 | DDG69	DDG56 | DDG1000
3	AB1000	SAG1001	CG54 | DDG69 | DDG1000	CG67 | CG73 | DDG69
3	AB1000	SAG1002	UNABLE | UNABLE	UNABLE | UNABLE
3	AB1000	SAG1003	UNABLE | UNABLE | UNABLE	UNABLE | UNABLE | UNABLE
3	AB1000	TBMD1001	CG54 | DDG54	CG54 | CG67
3	WP4000	TBMD4003	UNABLE | UNABLE	DDG59 | DDG69
3	WP4000	Escort4002	UNABLE | UNABLE | UNABLE	CG67 | DDG54 | DDG56

Figure 3. Mission Load Planner solve assignments page. In the ninth row, Mission Escort14001 has been assigned to DDG54, DDG69, and DDG1000 for Cycle1 of Warplan WP4000. During Cycle2, CG73, DDG59, and DDG1000 are assigned to this mission. Some missions here cannot be assigned to a ship that is designated UNABLE.

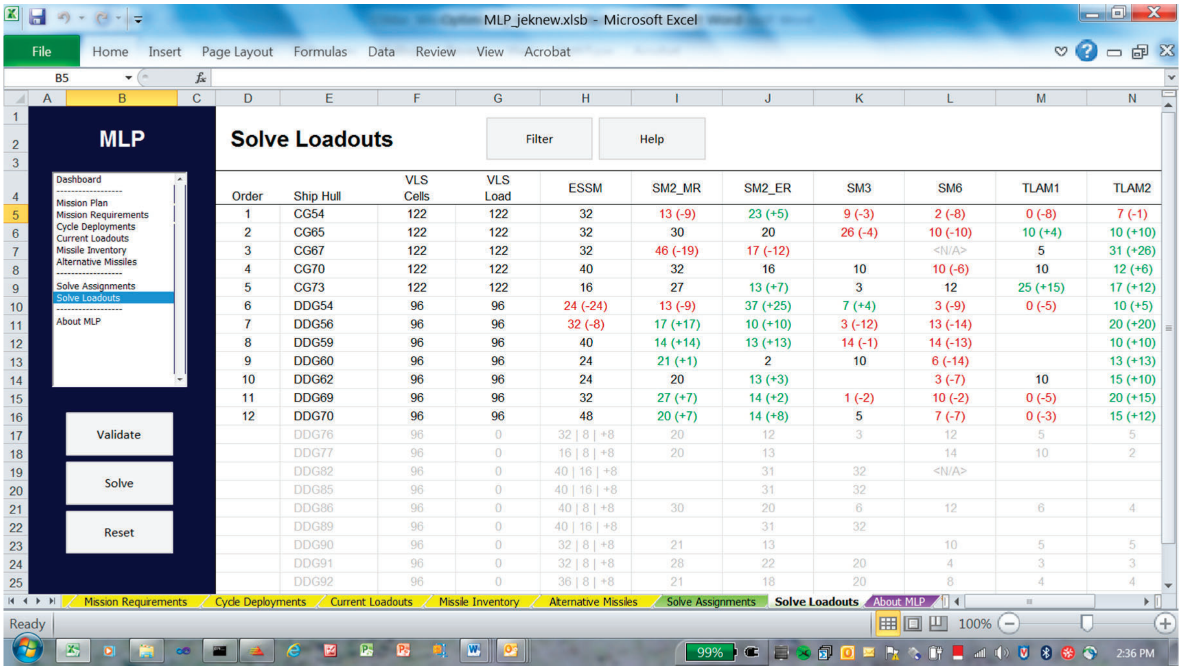


Figure 4. Solve loadouts page. This page shows ship hull number, VLS cells available, VLS load the cells used by our plan, and the numbers of each missile type loaded. The red (-) and green (+) fields, respectively, show reductions from and additions to entries in the "current loadouts" page (not shown). These suggested numbers of changes can be controlled by the Navy Mission Planner, ranging from none to optimal. An achievable revision is between these extremes.

accomplishment and the movement of missiles among ships and depot maintenance. This analysis is of interest to senior leadership and was prohibitively difficult before MLP.

MLP penalties can also be used to distribute missiles equitably among ships operating together. This is one of the key requirements for Tomahawk missile assignments (Newman et al., 2011).

The overarching planning problem is you never have enough of everything. This is why MLP offers so many policy penalties—rheostat knobs—to investigate the best we can do with what we do have under different decision criteria.

CONCLUSION

We have been asked to apply MLP to some real scenarios to support official Navy force structure alternative analyses. These involve equipping amphibious ships with new strike capability and investigating improvement in specific mission areas related to expeditionary warfare. The flexibility of the loadout planner's Excel data entry allows for easy modification of ship characteristics and her ability to complete new mission sets. MLP quantitatively assesses the value of the proposed improvements. Solve times are fast enough to allow same-hour responses to "what if" questions posed by the study's sponsor. In addition to the planner's original purpose of advising each deployer's missile loadout, the necessity of assigning warplan missions to a ship based on its best loadout to contribute to a warplan gives the planner additional capability to assess new capabilities and/or new mission areas. Finally, it is reassuring to be able to say, "Here is what we did, how we did it, and your answers," all completely documented without any hidden, unstated assumption.

Acknowledgements

At the Naval Postgraduate School, we are fortunate to have many domain experts in military and optimization modeling who are generous with their time and advice. We smile and remember Wayne Hughes advising, "Whatever you have loaded, to win, shoot first." Anton Rowe built our Excel user interface. Navy

Lieutenant Michael Wiederholt collected a lot of performance data and he and following student Navy Lieutenant Joseph Newman exercised our models, assessed outcomes, and suggested improvements. Matt Carlyle advised much of the VBA mimicry of a mathematical programming language to support heuristic solutions and reviewed an earlier draft.

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